Distributed Queue and Power Control for Rechargeable Sensor Networks under the SINR Interference Model

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***Abstract*—Renewable energy sources can be attached to sensor nodes to substantially improve the performance of sensor networks. In networks with renewable energy sources, conservative energy expenditure may lead to missed recharging opportunities due to the batteries being full, while aggressive usage of energy may cause the network to be intermittently disconnected and reduction in coverage. Thus, new techniques and algorithms must be developed for sensor networks with replenishment to balance these seemingly contradictory goals. In this paper, we consider a sensor network with renewable energy sources under the SINR interference model, where the achievable rate of each link depends on its SINR. We develop a provably efficient and distributed solution to maximize the total sensing rate subject to quality of service constraints. Our solution is a joint rate control, power allocation, and routing algorithm which is computationally simple and applicable to non-rechargeable sensor networks as well.**

I. Introduction

Wireless sensor networks have many applications in monitoring, maintenance, and environment sensing [1], [2], [3]. New developments in the area of renewable energy [4], [5] will help facilitate earlier and more wide-spread deployment of these networks by increasing their operational capacity and lifetime. These renewable sources of energy could be attached to the nodes, and would typically replenish the batteries at a slow rate (e.g., compared to the rate at which energy is consumed by a continuous stream of packet transmissions) that could be variable and dependent on the surroundings.

Unlike traditional battery-limited sensor networks, in networks with replenishment, the network lifetime can be infinity, and a node with a full battery may not be necessarily desirable. This is because a node with a full battery implies missed recharging opportunities, which in turn implies a reduced feasible sensing and transmission rates. On the other hand, an over aggressive use of energy may lead to lack of coverage or connectivity for certain time periods, or result in the network being incapable of transferring time-sensitive data to the sink because certain nodes have depleted their batteries. Thus, new techniques and algorithms must be developed for networks with replenishment to balance these seemingly contradictory goals. In this paper, we develop a distributed joint rate control, power allocation and routing algorithm, which is provably efficient for rechargeable networks. We use the so-called "physical interference model," for which the transmission rate is a function of the signal to interference and noise ratio (SINR) and influenced by the power vector over the entire network.

While the problem of energy management in wireless networks has seen considerable attention, there have been few works [6], [7], [8], [9], [10], [11] that also include energy replenishment. In [6], [7], [8], [9] the authors study the problem of dynamic node activation, energy harvesting, sleep-wake scheduling, and duty cycle controlling respectively under rechargeable networks. In [10] and [11], motivated by [12], the authors consider jointly managing the data and battery buffers to maximize the throughput with renewable energy sources under protocol interference model. The coupling between the data and battery buffers is what makes the problem notoriously difficult to solve using standard optimization based approaches. For example, unlike [12], which utilizes the fact that a static allocation policy is optimal, it is not even clear whether an optimal policy would be static in a rechargeable network setting. Specifically, with a battery buffer, there is an additional energy constraint that the allocated energy should be within the battery state, and this constraint is even more difficult than the average power constraint. In [10], the authors consider infinite data buffer and finite battery buffer sizes. They assume that the replenishing process is i.i.d, and show that the probability of battery state being less than the peak power or close to the full battery state vanishes as the battery size grows with their algorithm.

All the prior work on networks with replenishing nodes rely on discrete interference models, in order to develop practical solutions to the optimal resource allocation problem. Despite their simplicity, discrete interference models could be poor approximations to the actual interference, observed in wireless networks. On the other hand, physical interference models such as the SINR model are known to more accurately capture the impact of the interference more closely. Prior work on resource allocation and utility optimization under the SINR model include [13], [14], [15], [16], [17], [18]. In [13], [14], [15], the authors formulate optimization problems with QoS constraints on SINR. In [16], [17], [18], the authors consider an optimal power allocation problem such that a utility function of SINR is maximized. They have assumptions such as high or low interference regime to avoid the non-convexity. None of these studies consider energy replenishment. In this paper, we consider the power allocation problem combined

with rate control, to achieve the maximal sensing rate with energy replenishment using the SINR model. We do not assume high or low interference when use the rate power function, and our solution is applicable to non-rechargeable wireless networks as well.

The main contributions of the paper can be summarized as follows:

- We develop a mathematical framework to handle the coupling between the data and battery queues that exists in networks with replenishment. Our mathematical framework allows for multihop, dynamic routing, and does not require future knowledge of the data and replenishment.
- Unlike existing works on utility maximization and network optimization, in our framework, we allow for a general arrival and replenishment process that does not require stationarity or ergodicity. This is especially important in such networks, because the replenishment profiles are typically non-stationary.
- We develop a fully distributed, computationally simple, algorithm that achieves a provable constant fraction of the optimum solution under the SINR interference model. Note that the development of provably efficient distributed algorithms under the SINR interference model has been an open problem even for networks without replenishment.

II. System Model and Problem Formulation

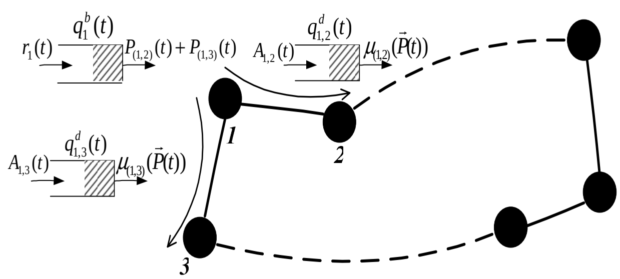


Fig. 1. Multihop Network Model

We consider a rechargeable wireless sensor network with N nodes and L links, as illustrated in Figure. 1. Each node $n \in \mathcal{N} = \{1, 2, \ldots, N\}$ is attached to a renewable power source for replenishment. We assume a time slotted system. In time slot t, $A_{n,e}(t)$ and $R_{n,e}(t)$ denote the amount of available data for sensing and the actual amount of sensed data, at node n that are destined to node e. Hence, $R_{n,e}(t) \leq A_{n,e}(t)$, $\forall t \geq 0$, $\forall n, e \in \mathcal{N}$. Data flows may traverse multiple hops to the destination and their routes are not fixed. Let $r_n(t)$ denote the amount of replenishment at node n in slot t. We assume that $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} A_{n,e}(t) \leq \lambda_{\max}$, $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} A^2_{n,e}(t) \leq A^2_{\max}$, $(0 < \lambda_{\max} < \infty,\ 0 < A^2_{\max} < \infty)$ and $0 < r_n(t) \leq r_{\max}$, $(0 < r_{\max} < \infty)$, $\forall t \geq 0$, $\forall n, e \in \mathcal{N}$. Without loss of generality, we assume the instantaneous amount of data $A_{n,e}(t)$ and replenishment $r_n(t)$ are available at the end of slot t. We assume that each node n maintains a separate infinite data buffer with state $q^d_{n,e}(t)$ for the flow destined to e, and also maintains a finite battery buffer with size $B^b_n < \infty$ and state $q^b_n(t)$. Note that we focus on infinite data buffer size and finite battery buffer size for ease of illustration. Following our approach in [11], we can also allow for finite battery and data buffers. We assume that, for each link $l \in \mathcal{L} = \{1, 2, \ldots, L\}$, the power used to transmit a packet by the transmitting node and the power used to receive and decode a packet by a receiving node are identical and equal to $P_l(t)$. Note that this assumption is merely for notational convenience and the generalization to the asymmetric case is straightforward. Vector $\vec{P}(t) = \{P_1(t), P_2(t), \ldots, P_L(t)\}$ denotes the power assignments over all the links at time t. With this power allocation, the achieved data rate at link l is $\mu_l(\vec{P}(t))$ in that time slot. Let $\vec{P}$ be the sequence of power assignments $\{\vec{P}(0), \vec{P}(1), \ldots, \vec{P}(T-1), \ldots\}$ and $\vec{R}$ be the sequence of sensing rates $\{\vec{R}(0), \vec{R}(1), \ldots, \vec{R}(T-1), \ldots\}$. Let Ω_n denote the set of directed links that originate from node n, and let Θ_n denote the set of directed links that terminate at node n. We assume that the total power used by a node in each slot is upper bounded by the peak power P_{peak} of sensor hardware, i.e., $\sum_{l\in\Omega_n \bigcup \Theta_n} P_l(t) \leq P_{\text{peak}}$. The rate-power function is assumed to have the following standard form under the SINR interference model:

$$\mu_l(\vec{P}(t)) = \log\left(1 + \frac{g_{\text{tran}(l),\text{rec}(l)} P_l(t)}{N_{\text{rec}(l)} + \sum_{i\in\mathcal{L}, i\neq l} g_{\text{tran}(i),\text{rec}(l)} P_i(t)}\right)$$
$$= \log\left(1 + \frac{P_l(t)}{\sigma_l + \sum_{i\in\mathcal{L}, i\neq l} h_{i,l} P_i(t)}\right) \quad (1)$$

where $g_{\text{tran}(i),\text{rec}(j)}$ is the channel gain from the transmitting node tran(i) of link i, to the receiving node rec(j) of link j, N_n is the background noise at node n, $\sigma_l = \frac{N_{\text{rec}(l)}}{g_{\text{tran}(l),\text{rec}(l)}}$ is the noise to gain ratio of link l. Note that, even though $\log(\cdot)$ is a concave function, $\mu_l(\vec{P}(t))$ is *not* a concave function of $\vec{P}(t)$. We can see from this rate-power function that the transmission rate on link l is influenced by the power assignment on all other links over the entire network (interference) based on the SINR model.

Definition 1: p^o_n is the *long-term battery discharge ratio* for node n:

$$p^o_n = \limsup_{T\to\infty} \frac{\sum_{t=0}^{T-1} I^o_n(t)}{T} \quad (2)$$

where

Definition 2:

$$I^o_n(t) = \textit{indicator that battery is discharged in slot t for node n}$$
$$= \begin{cases} 0 & \text{if } \sum_{l\in\Omega_n \bigcup \Theta_n} P_l(t) < q^b_n(t) \\ 1 & \text{otherwise} \end{cases} \quad (3)$$

Definition 1 is a suitable QoS metric for rechargeable sensor networks that represents how often the battery is discharged. Some applications may require the battery state to be always above certain positive level, then Definition 1 can be easily

modified to represent how often the battery is below the desired level, and our solution structure works as well. Note that the ratio p_n^o represents the actual probability, if the system variables were to satisfy the strong law of large numbers (SLLN), which we do not assume here. From a quality of service perspective, it is desirable to keep p_n^o to be less than some small upper bound η_n^o.

Our overall objective is to maximize the long-term average total sensing rate at all nodes destined to all destinations subject to the constraints on the "rate" of battery discharge for all nodes. With the above setting, we formulate the problem as follows:

$$(A) \qquad \max_{\vec{P},\vec{R}} \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\sum_{n,e\in\mathcal{N}} R_{n,e}(t)$$

$$q_{n,e}^d(t+1) \le \Big(q_{n,e}^d(t) - \sum_{l\in\Omega_n} \mu_l^e\big(\vec{P}(t)\big)\Big)^+ + R_{n,e}(t) + \sum_{l\in\Theta_n} \mu_l^e(\vec{P}(t)), \quad n \neq e, \tag{4}$$

$$q_n^b(t+1) = \min\left[q_n^b(t) - \sum_{l\in\Omega_n\cup\Theta_n} P_l(t) + r_n(t), B_n^b\right], \tag{5}$$

$$0 \le \sum_{l\in\Omega_n\cup\Theta_n} P_l(t) \le \min\Big[q_n^b(t), P_{\text{peak}}\Big], \tag{6}$$

$$\sum_{e=1}^{N} \mu_l^e\big(\vec{P}(t)\big) = \mu_l\big(\vec{P}(t)\big), \quad R_{n,e}(t) \le A_{n,e}(t), \tag{7}$$

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q_{n,e}^d(t) < \infty, \quad n \neq e, \tag{8}$$

$$p_n^o \le \eta_n^o, \quad \forall n \in \mathcal{N}. \tag{9}$$

In Problem (A), constraints (4) and (5) describe how the data[1] and battery queues evolve, respectively. Constraints (6) are the energy conservation equations ensuring that we cannot oversubscribe the energy that is unavailable in the battery, nor can we exceed the peak power level. Constraints (7) are the rate conservation equations that bound the actual amount of sensed data $R_{n,e}(t)$ by the available amount of data $A_{n,e}(t)$, and share the transmission rate of a link among all the destinations in slot t. Constraint (8) is the QoS constraint for the data queue: we need to keep all the data queues stable. Constraint (9) is the QoS constraint that guarantees a battery discharge rate of η_n^o.

We proceed along the following lines:

- We define *virtual queues* for the battery buffers. We show that keeping these virtual queues stable ensures that Constraint (9) is met.
- We design a distributed algorithm that keeps the data queues and virtual battery queues stable, and at the same time achieves a constant fraction of the global optimum of Problem (A) within a provably small gap.

[1]Note that if the physical layer does not allow full duplex operation, $g_{n,n}$ can be chosen to be infinite, so that the node will not be able to transmit and receive at the same time.

Definition 3: We define *virtual battery queues* $\forall n \in \mathcal{N}$:

$$\tilde{q}_n^b(t+1) = \Big(\big(\tilde{q}_n^b(t) - \eta_n^o\big)^+ + I_n^o(t) - r_n(t) + M_n(t) + \sum_{l\in\Omega_n\cup\Theta_n} P_l(t)\Big)^+, \tag{10}$$

where $M_n(t) = \big(q_n^b(t) - \sum_{l\in\Omega_n\cup\Theta_n} P_l(t) + r_n(t) - B_n^b\big)^+$ is the amount of missed replenishment and $I_n^o(t)$ are defined in Equation (3).

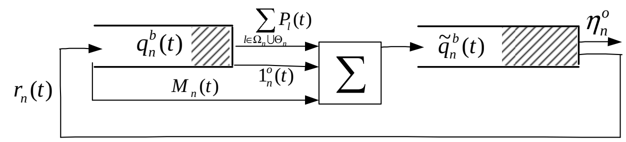


Fig. 2. Battery Queue and Virtual Battery Queue

From Figure 2, $\sum_{l\in\Omega_n\cup\Theta_n} P_l(t) - r_n(t) + M_n(t)$ is actually the decrement of $q_n^b(t)$ from slot t to $t+1$. Since the battery has finite size, this term vanishes on average as time goes to infinity. Then, $p_n^o = \limsup_{T\to\infty} \frac{\sum_{t=0}^{T-1} I_n^o(t)}{T}$ and η_n^o can be viewed as the long-term input and output rate of $\tilde{q}_n^b(t)$, respectively. Thus, it is reasonable to expect that $\tilde{q}_n^b(t)$ being stable implies $p_n^o \le \eta_n^o$.

Theorem 1: If all the virtual battery queues $\tilde{q}_n^b(t)$ are strongly stable, i.e., $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \tilde{q}_n^b(t) < \infty$, $\forall n \in \mathcal{N}$, we then have $p_n^o \le \eta_n^o$, $\forall n \in \mathcal{N}$.

The proof can be found in Appendix A. We give the main algorithm in the following section.

III. Joint Rate Control, Power Allocation and Routing Algorithm for Multihop Networks

We begin by defining the notion of crosstalk.

Definition 4:

$$h_{i,j} = \frac{g_{\text{tran}(i),\text{rec}(j)}}{g_{\text{tran}(j),\text{rec}(j)}}$$

is the *crosstalk* between a given link i to another link j.

From Equation (1), the rate on any link l is a function of the power allocation over all the links in the network. To localize the power control, our power allocation scheme activates, i.e., allocates a non-zero power only to links with crosstalks under a certain threshold $H > 0$.

Definition 5: The *conflict set* $\mathcal{E}_l$ of link l is defined as follows:

$$\mathcal{E}_l = \{k \in \mathcal{L} : \ k \neq l; \ h_{k,l} > H \text{ or } h_{l,k} > H\}$$

We will discuss how we choose H later. If link l is activated, the links in its conflict set $\mathcal{E}_l$ can not be activated. Note that for any two links i and j, if $i \in \mathcal{E}_j$, then $j \in \mathcal{E}_i$.

Definition 6: The *conflict degree* $K(l)$ of link l is the maximum number of links in $\mathcal{E}_l$ that can be activated simultaneously. The conflict degree K of the network is defined as

follows:

$$K = \max_{l} K(l).$$

The notion of conflict degree is similar to the interference degree under protocol interference model [19]. We use the term "conflict" here because even links that are not in each other's conflict list can still interfere with each other under the SINR model.

Definition 7: A set of links S is a *maximal schedule* if all the links are not conflicting with each other, but adding any link outside S will result in a conflict.
Note that $\sum_{k\in l\bigcup\mathcal{E}_l} I_{[k\in S]} \geq 1,\ \forall l \in \mathcal{L}$.

Our algorithm is composed of three components: rate control, power allocation, and routing.

Multihop Rate Control (MRC):
If $q^d_{n,e}(t) \leq \frac{KV}{2}$, node n chooses to sense all the available data until reach a given bound W, i.e., $R_{n,e}(t) = \min[A_{n,e}(t), W]$; otherwise, reject all the data, i.e., $R_{n,e}(t) = 0$. Here, $V, W \in (0, \infty)$.

Multihop Power Allocation (MPA):
Here the goal is to ensure that no node transfers data of a flow to a relay node that is not the destination of that flow, unless the differential backlog for that flow is greater than a fixed value $\gamma > 0$. We will choose the value of γ such that the resulting backlog of the receiving node is not larger than that of the transmitting node after the transmission. This pushes the data flow from the source to the destination with a positive back pressure. We then define

$$\gamma^e_l = \begin{cases} \gamma & \text{if rec}(l) \neq e \\ 0 & \text{otherwise} \end{cases}$$

Recall that tran(l) and rec(l) denote the transmitting and receiving node of link l, respectively. Let $e_l(t) = \arg\max_e \left\{q^d_{\text{tran}(l),e}(t) - q^d_{\text{rec}(l),e}(t) - \gamma^e_l\right\}$ be the flow on link l that has the maximal modified differential backlog, and $w_l(t) = \max\left[q^d_{\text{tran}(l),e_l(t)}(t) - q^d_{\text{rec}(l),e_l(t)}(t) - \gamma^{e_l(t)}_l, 0\right]$ is the nonnegative differential backlog of l at time t.

The network randomly generates a maximal schedule $S(t)$ in a fully distributed manner as in [19], [20], and solve

$$\begin{aligned} \max_{\vec{P}(t)} & \sum_{l\in S(t)} \left[w_l(t)\mu_l\big(P_l(t)\big) - \Big(\tilde{q}^b_{\text{tran}(l)}(t) + \tilde{q}^b_{\text{rec}(l)}(t)\Big)P_l(t)\right] \\ \text{s.t.} \quad & 0 \leq \sum_{l\in\Omega_n\cup\Theta_n} P_l(t) \leq \min\left[q^b_n(t), P_{\text{peak}}\right], \end{aligned} \tag{11}$$

where $\mu_l\big(P_l(t)\big) = \log\left(1 + \frac{P_l(t)}{\sigma_l}\right)$. The objective function is separable and concave, and the constraints are local and linear, so this optimization problem can be solved in a distributed manner [21].

Multihop Routing:
When $w_l(t) > 0$, transmit for flow that is destined to $e_l(t)$ with rate $\mu_l\big(\vec{P}(t)\big)$, i.e., $\mu^{e_l(t)}_l\big(\vec{P}(t)\big) = \mu_l\big(\vec{P}(t)\big)$ and $\mu^e_l\big(\vec{P}(t)\big) = 0,\ \forall e \neq e_l(t)$.
It is apparent that *MRC* and routing can be applied by each node independently.

Observation 1: From the objective function in Equation (11), $w_l(t)\mu_l\big(P_l(t)\big)$ is the utility of link $l \in S(t)$, when $w_l(t)$ is large, i.e., the differential backlog of the data queues is large, *MPA* tends to allocate a larger value of $P_l(t)$ so that $q^d_{\text{tran}(l),e_l(t)}(t)$ will decrease. Further, the routing component is used to decrease the length of all data queues over all flows.
Observation 2: $(\tilde{q}^b_{\text{tran}(l)}(t)+\tilde{q}^b_{\text{rec}(l)}(t))P_l(t)$ is the cost of link l, once $q^b_{\text{tran}(l)}(t)$ or $q^b_{\text{rec}(l)}(t)$ is discharged, $\tilde{q}^b_{\text{tran}(l)}(t)$ or $\tilde{q}^b_{\text{rec}(l)}(t)$ tends to increase by Equation (10), then *MPA* allocates smaller $P_l(t)$ so that the discharge event happens infrequently.
Observation 3: From Figure. 2, as $q^b_{\text{tran}(l)}(t)$ or $q^b_{\text{rec}(l)}(t)$ increases, $\tilde{q}^b_{\text{tran}(l)}(t)$ or $\tilde{q}^b_{\text{rec}(l)}(t)$ decreases, then *MPA* tends to allocate larger $P_l(t)$ to avoid battery overflow in advance.
From *Observation 2* and *Observation 3*, *MPA* expends energy neither too conservatively nor too aggressively.

Note that the amount of actual sensing data $R_{n,e}(t)$ is bounded and the endogenous arrivals to any node is also bounded due to the peak power constraint and interference. Then it is clear that by using the *MRC*, we make sure that the data queues remain within a certain bound and this has a positive effect on the batteries as well, since a portion of data is not allowed into the nodes. The natural question one would ask here is, whether *MRC* rejects too many packets in the first place to *synthetically* meet the constraints. In the following theorem, we show that this is not the case. Indeed, if there exists a feasible solution, $\lambda^* = \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{n,e} R^*_{n,e}(t)$ to Problem (A) for a given $\{\vec{A}(t),\ t\geq 0\}$ and $\{\vec{r}(t),\ t \geq 0\}$, then the sensing rate associated with *MRC* and *MPA* can be made asymptotically close to λ^* by appropriately choosing the control parameters V and W as the size of batteries B^b_n grows. We use the notation $y = O(x)$ to represent y going to 0 as x goes to 0.

Theorem 2: If a feasible solution to Problem (A) exists and the optimal instantaneous sensing rate vector is $\vec{R}^*(t)$, then the joint rate control *MRC*, power allocation *MPA*, and multihop routing algorithm achieves:

$$q^d_{n,e}(t) \leq \frac{K}{2}V + W, \tag{12}$$

$$\tilde{q}^b_n(t) \leq \frac{K}{2}V + W, \tag{13}$$

$$\begin{aligned} &\sum_n \left\{\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_e R_{n,e}(t)\right\} \\ &\geq \sum_n \left\{\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_e \frac{R^*_{n,e}(t)}{K} - \eta^o_n\big(2+\mu_{\max}\big) - \epsilon(W)\right. \\ &\quad \left. - O\Big(\frac{(\frac{K}{2}V + W - B^b_n)^+}{V}\Big) - \sum_{l\in\Omega_n}\frac{P^2_{\text{peak}}LH}{\sigma^2}\right\} - O(\frac{W}{V}). \end{aligned} \tag{14}$$

where $\sigma = \min_l \sigma_l$ and $\mu_{\max} = \log(1 + \frac{P_{\text{peak}}}{\sigma})$.
The detailed proof is in Appendix B. Now, we discuss the implications of this theorem.

In Theorem 2, V is a finite tunable approximation parameter, which controls the efficiency of the algorithm. W and H

are algorithmic parameters, and B_b is the system parameter.

Implication 1: W is a parameter in *MRC* used to bound the actual sensing rate. The term $\epsilon(W)$ can be arbitrarily small by choosing W large enough.

Implication 2: From Equation (12) and Equation (13), the data queues $q^d_{n,e}(t)$ and virtual queues $\tilde{q}^b_n(t)$ are all strongly stable. Thus, by Theorem 1, $p^o_n \leq \eta^o_n$. Given W, as V increases, the term $O(\frac{W}{V})$ in the gap decreases, but the average data queue length and virtual battery queue length tend to increase, which implies that the queueing delay may increase.

Implication 3: Equation (14) compares the performance of our algorithm with that of the optimal solution of Problem (A). The term $\sum_n \eta^o_n(2+\mu_{\max})$ captures the influence of network disconnection due to battery discharges, and it is small since η^o_n is typically chosen to be very small for all n.

Implication 4: Given W, the terms $O(\frac{(\frac{K}{2}V+W-B^b_n)^+}{V})$ and $O(\frac{W}{V})$ together capture the influence of battery overflow and represents the asymptotic behavior of the gap. By choosing V and B^b_n to be large, we can make the performance gap decay inversely proportional to the buffer sizes. A larger battery size implies smaller amount of wasted replenishing opportunities, which is consistent with intuition.

Implication 5: The term $\frac{P^2_{\text{peak}}LH}{\sigma^2}$ captures the performance loss due to imperfect scheduling based on conflict sets. This term decreases as H decreases. It is apparent that $K \leq L < \infty$, so the fraction $\frac{1}{K}$ is a constant fraction. However, L is a very loose bound for K in large networks (large L). From Equation (14), we would like both K and H to be small, so the question is by decreasing H, whether K will increase to a very large value. The following example gives a bound for K in a large deterministic uniform network.

Example: *Uniform Extended Network*

In this example, we consider a uniform extended network, where nodes are distributed deterministically at a density ρ, homogenously within the network. Let $d(m,n)$ be the distance between node m and node n, $m,n \in \mathcal{N}$. Let $d_l = d(\text{tran}(l), \text{rec}(l))$, $d_{\max} = \max_{l\in\mathcal{L}} d_l$, $d_{\min} = \min_{l\in\mathcal{L}} d_l$ and $\xi = \lfloor \frac{d_{\max}}{d_{\min}} \rfloor$. Let

(1) $0 < d_{\min} \leq d_{\max} < \infty$,

(2) there exist $\alpha, \beta > 0$ such that $g_{m,n} = \alpha d^{-\beta}(m,n)$, $\forall m,n \in \mathcal{N}$, i.e., signal power decays with distance with a path loss exponent β.

Theorem 3: For the uniform deterministic extended network described above,

$$K \leq \max\left\{4\pi\rho d^2_{\max}, 6+\xi\left\lfloor\frac{2\pi}{\arccos\left(1-\frac{d^2_{\min}}{2d^2_{\max}}\right)}\right\rfloor\right\}.$$

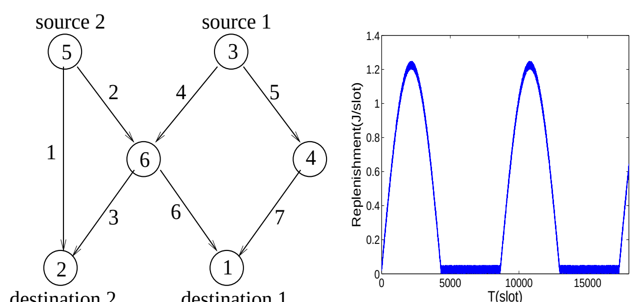
The proof of Theorem 3 can be found in Appendix C. From Theorem 3, we can see that K is upper bounded by a constant which is not a function of H. The intuition is: as H decreases, the conflict area of link l becomes larger and the number of links in $\mathcal{E}_l$ increases. However, the conflicting opportunities of links in $\mathcal{E}_l$ also increases. Thus, the overall impact of these two factors cancel each other and K remains bounded.

IV. Numerical Examples

We consider a network topology, shown in Figure 3(a). There are 6 nodes, 7 links, and 2 flows with source-destination pair $(3,1)$ and $(5,2)$, respectively. We use the rate power function as in Equation (1). We choose the power vector of the background noise $[N] = [2,2,2,2,2,2]\times 10^{-14}W$, and the channel gain matrix

$$[\mathbf{g}] = \begin{pmatrix} \infty & 0.01 & 0.01 & 2 & 0.01 & 2 \\ 0.01 & \infty & 0.01 & 0.01 & 0.01 & 2 \\ 0.01 & 0.01 & \infty & 2 & 0.01 & 2 \\ 2 & 0.01 & 0.01 & \infty & 0.01 & 0.01 \\ 0.01 & 2 & 0.01 & 0.01 & \infty & 0.1 \\ 2 & 0.1 & 0.01 & 0.01 & 2 & \infty \end{pmatrix} \times 10^{-13},$$

and hence $\sigma = 0.1$. The number of arrivals $A_{3,1}(t),\ t \geq 0$ and $A_{5,2}(t),\ t\geq 0$, are modeled as independent Poisson random variables with mean $\lambda = 20$ packets/slot and $A_{\max} = 30$ packets/slot. We set η^o_n, the threshold of battery outage probability to 0.03 for all $n \in \mathcal{N}$ and the peak power $P_{\text{peak}} = 1.5W$. The backlog threshold $\gamma = 80 \geq \max_{n,e}\left(\sum_{\text{rec}(l)=n}\mu_l + A_{n,e}\right) = 2\mu\left(1+\frac{P_{\text{peak}}}{\sigma}\right)$, so that the resulting backlog of the receiving node is not longer than that of the transmitting node. In all simulations, the simulation time is $T = 10^6$ time slots.



(a) Network Topology (b) Replenishment Process

Fig. 3. Network topology and a sample replenishing process

Scenario 1: We first use a replenishment process which is formed by a periodic deterministic sine waveform ($r_{\max} = 1.2$ and period 8000) plus independent Gaussian noise with mean 0.01, as shown in Figure 3(b) (The cycles imitate the daily solar cycles for a solar battery and the average replenishing can be simply calculated $\bar{r} = 0.2$). All the battery buffer sizes are set to be $B^b = 300J$. We simulate three cases: (1) crosstalk threshold $H = 1$ (the notion of H first appear in Definition 5), then we obtain $K = 3$ and there are 7 possible schedules; (2) $H = 0.05$ (H cannot be too small, otherwise a link may conflict with all other links. In that case, we may need to increase V to a very large value to guarantee a large sensing rate, which leads to large queue length and delay on the other hand). With this choice, $K = 3$ (we can see that K does not increase as H decreases) and there are 4 possible schedules; (3) Use the maximal schedules under the node exclusive model, then $K = 2$ and there are 6 possible schedules. We choose different values of the control coefficient V for the proposed algorithm and compare the results with the

optimal value[2]. From Figure 4 (a), we see that as V increases, the average total sensing rate keeps increasing and gets closer to a value that is much larger than $\frac{1}{K} = \frac{1}{3}$ (or $\frac{1}{2}$ for case (3)) of the optimal value, which is consistent with Equation (14). Case (2) outperforms case (1) since it has smaller H. It is not surprising that case (2) also outperforms case (3), since case (3) choose schedules according to a protocol model, and it results in more schedules than case (2). Time sharing of two schedules[2] (these two schedules are contained in all three cases) gives the best performance, so the case that has less possible schedules should perform better. From Figure 4 (b), we see that as V increases, the average data queue length (we here only plot the data queue length of node 3 for flow 1 due to space limitation) keeps increasing but is upper bounded by the bound we get in Equation (12). Case (3) has the best delay performance, i.e., less average queue length since it has smaller K. From Figure 4 (c) we observe that the battery discharge probability (we only plot for node 3 here) increases to the threshold as V increases. Case (3) goes to the threshold slower that (1) and (2) since it has less K which results in less virtual battery queue length by Equation (13). Case (2) outperforms case (1) in the sensing rate but the discharge ratio increases faster, which shows the tradeoff by increasing V.

Scenario 2: We use a different replenishing processes: $r_2(t)$ and $r_5(t)$ are i.i.d binomial random variables *Binomial*$(1, 0.5)$; replenishing at all other nodes are independent binomial random variables $0.2 \times$ *Binomial*$(1, 0.5)$ in even number slots, and $0.6 \times$ *Binomial*$(1, 0.5)$ in odd number slots ($\bar{r}_2 = \bar{r}_5 = 0.5$ and $\bar{r} = 0.2$ for other nodes), all plus Gaussian noise with mean 0.03. We set $H = 0.05$ and simulate[2] three different battery sizes $B^b = 100J$, $B^b = 10J$ and $B^b = 1J$ (all nodes has the same battery sizes). From Figure. 4 (d), we can see that the sensing rate increases as battery size increases. However, as long as the battery is large compared to the average replenishing rate, the improvement diminishes with increasing battery sizes.

V. Conclusion

In this paper, we studied the problem of energy management in rechargeable wireless sensor networks under SINR interference model. Our objective was to maximize the average data sensing rate subject to QoS constraints on both data and battery queues. We developed a mathematical framework that allows for multi-hop dynamic routing and handles handle the coupling between the data and battery queues. Unlike existing works, we allow for a the practical realities of having non-stationary replenishment and arrival processes. We develop a fully distributed, computationally simple, algorithm that achieves a provable constant fraction of the optimum solution under the SINR interference model. Through simulations, we show the efficacy of our solution. The impact of fairness and channel variations (fading model) can be easily accommodated into the mathematical formulation. However, an open question for future work will be to develop distributed algorithms that are provably efficient under a fading model.

[2] The exact optimal value for Problem (A) is hard to obtain, so we here use an upper bound for the optimum that is calculated under the interference free environment. For this example, an upper bound for the optimum can be obtained by equal time sharing of schedules $\{1, 4, 7\}$ and $\{1, 5, 6\}$, and utilize the interference free link rate $\mu(\bar{r})$ under infinite buffer and no discharge constraint. In *Scenario 1*, the optimum is $2\mu(\bar{r}) = 30.4$. In *Scenario 2*, since $\mu(\bar{r}_2) = \mu(\bar{r}_5) > \lambda$, the optimum is $\lambda + \mu(\bar{r}_3) = 35.2$

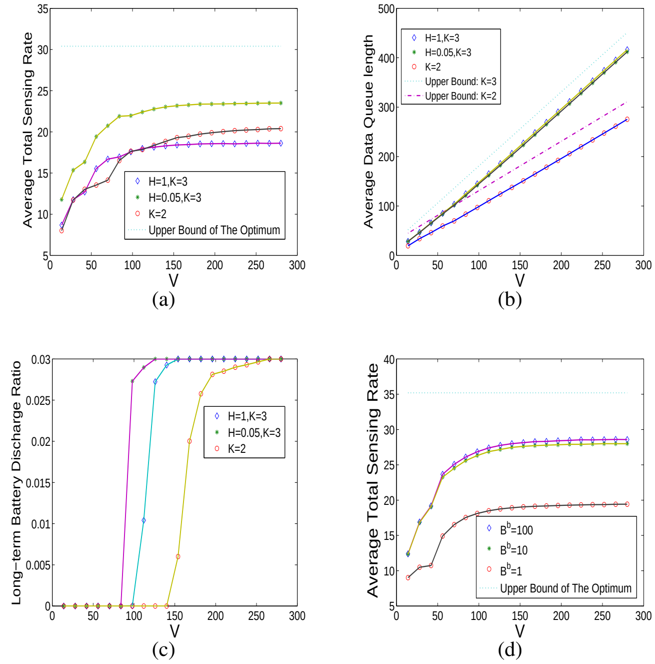


Fig. 4. Impact of the control parameter V and threshold H on (a) the average total sensing rate, (b) average data queue length of $q^d_{3,1}$, and (c) the battery discharge probability of node 3 for *Scenario 1*. Impact of battery size on (d) the average total sensing rate for *Scenario 2*.

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Appendix

A. Proof of Theorem 1

Using the idea similar to [12], we have the fact that if any queue represented with $Q(t)$ is strongly stable, then $\limsup_{T\to\infty}\frac{Q(T)}{T}\le 0$. Hence, if $\tilde{q}_n^b(t)$ is strongly stable, then $\limsup_{T\to\infty}\frac{\tilde{q}_n^b(T)}{T}\le 0$. From Equation (10), we have $\tilde{q}_n^b(t+1)\ge \tilde{q}_n^b(t)-\eta_n^o+\sum_{l\in\Omega_n\bigcup\Theta_n}P_l(t)-r_n(t)+M_n(t)+I_n^o(t)$. Note that $q_n^b(t+1)=q_n^b(t)-\sum_{l\in\Omega_n\bigcup\Theta_n}P_l(t)+r_n(t)-M_n(t)$. By summing from 0 to $T-1$, dividing by T and taking $\limsup$ of both sides, we have

$$\limsup_{T\to\infty}\frac{\tilde{q}_n^b(T)}{T}\ge\lim_{T\to\infty}\frac{\tilde{q}_n^b(0)}{T}-\eta_n^o+\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}I_n^o(t)+\lim_{T\to\infty}\frac{q_n^b(0)-q_n^b(T)}{T}.$$

Since $\limsup_{T\to\infty}\frac{\tilde{q}_n^b(T)}{T}\le 0$, $\limsup_{T\to\infty}\frac{\tilde{q}_n^b(0)}{T}=0$ $\limsup_{T\to\infty}\frac{q_n^b(T)}{T}=0$, and $\lim_{T\to\infty}\frac{\tilde{q}_n^b(0)}{T}=0$, so we get $p_n^o=\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}I_n^o(t)\le\eta_n^o,\ \forall n\in\mathcal{N}$. ∎

B. Proof of Theorem 2

Proof of Equation (12): We prove Equation (12) by induction. Let $q_{\max}^d(t)$ be the maximum data queue length for all flows at slot t. Assume that $q_{\max}^d(t)\le\frac{KV}{2}+W$ (holds for $t=0$ by letting $q_{n,e}^d(0)=0,\ \forall n,e\in\mathcal{N}$), need to show that it holds at slot $t+1$. Consider the data queue $q_{n,e}^d(t+1)$ maintained at any node n for flow destined to any node $e\neq n$ at slot $t+1$. If node n received data destined to e from other node m at slot t, then by the routing policy in Section III and definition of $w_{(m,n)}(t)$, $q_{m,e}^d(t)-q_{n,e}^d(t)>\gamma_{(m,n)}^e$, where (m,n) is the link from node m to node n. Choose γ such that the resulting backlog of the receiving node is not longer than that of the transmitting node (let $\mu_{\max}^{\text{in}}$ to be the maximum endogenous arrivals, then $\gamma=\mu_{\max}^{\text{in}}+W$ satisfy this condition), we then have $q_{n,e}^d(t+1)\le q_{n,e}^d(t)+\gamma$, then $q_{n,e}^d(t+1)\le q_{m,e}^d(t)\le q_{\max}^d(t)\le\frac{KV}{2}+W$. If node n did not receive any data destined to e from other nodes, then it can only have exogenous arrivals. Clearly $q_{n,e}^d(t+1)\le q_{n,e}^d(t)\le\frac{KV}{2}+W$ if there were no exogenous arrivals. If there were exogenous arrivals, by *MRC* of Section III, we must have $q_{n,e}^d(t)\le\frac{KV}{2}$, then $q_{n,e}^d(t+1)\le\frac{KV}{2}+W$. Thus, Equation (12) holds.

Proof of Equation (13): Proving Equation (13) from *MPA* is the same as the node exclusive interference model. We here prove for a more general case with the rate power function $\mu_l\big(\vec{P}(t)\big)$. Consider any link $l^*\in\mathcal{L}$, when the power assignment on other links are fixed, $\mu_{l^*}(\cdot)=\log(\cdot)$ is monotonically increasing, reversible, and differentiable on $P_l(t)$. Hence, we have $\mu_{l^*}\big(\vec{P}(t)\big)\le\mu_{l^*}\big(\vec{P}(t)-P_{l^*}(t)\vec{l^*}\big)+P_{l^*}(t)$ with fixed power on other links, where $\vec{l^*}$ is a vector of length L with 1 at the l^*-th index and 0 at others. Since $\vec{P}(t)-P_{l^*}(t)\vec{l^*}\in\Pi(t)$ is the resulting power vector by deactivating link l^* from $\vec{P}(t)$, if $\vec{P}(t)\in\Pi(t)$, $\vec{P}(t)-P_{l^*}(t)\vec{l^*}\in\Pi(t)$ also holds since the battery constraints will not be violated. Consider any node $n^*\in\mathcal{N}$ and any link $l^*\in\Omega_{n^*}$. Since $\mu_{l^*}\big(\vec{P}(t)\big)\le\mu_{l^*}\big(\vec{P}(t)-P_{l^*}(t)\vec{l^*}\big)+P_{l^*}(t)$ and $\mu_l\big(\vec{P}(t)\big)\le\mu_l\big(\vec{P}(t)-P_{l^*}(t)\vec{l^*}\big)$, $l\neq l^*$ (for link $l\neq l^*$, energy on l^* is its interference, then reduce the interference will increase the data rate), we then have

$$\begin{aligned}&\sum_l\Big[w_l(t)\mu_l\big(\vec{P}(t)\big)-\Big(\tilde{q}_{\text{tran}(l)}^b(t)+\tilde{q}_{\text{tran}(l)}^b(t)\Big)P_l(t)\Big]\\&\le\sum_l\Big[w_l(t)\mu_l\big(\vec{P}(t)-P_{l^*}(t)\vec{l^*}\big)-\Big(\tilde{q}_{\text{tran}(l)}^b(t)+\tilde{q}_{\text{tran}(l)}^b(t)\Big)\\&\quad\cdot P_l(t)\Big]+w_{l^*}(t)P_{l^*}(t).\end{aligned}$$

Further, since $\vec{P}(t)-P_{l^*}(t)\vec{l^*}\in\Pi(t)$, we have from *PA* that

$$\begin{aligned}&\sum_l\Big[w_l(t)\mu_l\big(\vec{P}(t)\big)-\Big(\tilde{q}_{\text{tran}(l)}^b(t)+\tilde{q}_{\text{tran}(l)}^b(t)\Big)P_l(t)\Big]\\&\ge\sum_l\Big[w_l(t)\mu_l\big(\vec{P}(t)-P_{l^*}(t)\vec{l^*}\big)-\Big(\tilde{q}_{\text{tran}(l)}^b(t)+\tilde{q}_{\text{tran}(l)}^b(t)\Big)\\&\quad\cdot P_l(t)I_{[l\neq l^*]}\Big].\end{aligned}$$

Thus, $\Big(\tilde{q}_{\text{tran}(l^*)}^b(t)+\tilde{q}_{\text{rec}(l^*)}^b(t)\Big)P_{l^*}(t)\le w_{l^*}(t)P_{l^*}(t)$. If all the incident links of a node is deactivated, then the virtual battery queue this node does not increase anyway. Without loss of generality, we only need to consider the case that $P_{l^*}(t)>0$. Therefore, $\tilde{q}_{\text{tran}(l^*)}^b(t)+\tilde{q}_{\text{rec}(l^*)}^b(t)\le w_{l^*}(t)\le q_{\max}^d(t)\le\frac{KV}{2}+W$. Since n^*,l^* can be arbitrary, we have $\tilde{q}_n^b(t)\le\frac{KV}{2}+W,\ \forall n\in\mathcal{N}$.

Proof of Equation (14): Define $L\big(\vec{q}^d(t), \vec{\tilde{q}}^b(t)\big) = \frac{1}{K}\sum_{n,e}\big(q^d_{n,e}(t)\big)^2 + \frac{1}{K}\sum_n\big(\tilde{q}^b_n(t)\big)^2$. From Equation (4) and Equation (10), we have

$$\begin{aligned}&\Delta(t) = L\big(\vec{q}^d(t+1), \vec{\tilde{q}}^b(t+1)\big) - L\big(\vec{q}^d(t), \vec{\tilde{q}}^b(t)\big)\\ &\leq \frac{1}{K}\sum_{n,e}\Big[A^2_{n,e}(t) + 2\mu_{\max}A_{n,e}(t) + 2\mu^2_{\max}\Big] + \frac{1}{K}\sum_n\Big[(1+\\ &P_{\text{peak}})^2 + \big(\eta^o_n\big)^2 + r^2_{\max} + 2\eta^o_n r_{\max}\Big] + \frac{2}{K}\sum_{n,e} q^d_{n,e}(t)R_{n,e}(t)\\ &-\frac{2}{K}\sum_l\Big[\Big(q^d_{\text{tran}(l),e_l(t)}(t) - q^d_{\text{rec}(l),e_l(t)}(t)\Big)\mu_l\big(\vec{P}(t)\big) -\\ &\Big(\tilde{q}^b_{\text{tran}(l)}(t) + \tilde{q}^b_{\text{tran}(l)}(t)\Big)P_l(t)\Big] + \frac{2}{K}\sum_n \tilde{q}^b_n(t)\Big[I^o_n(t) -\\ &r_n(t) + M_n(t)\Big].\end{aligned}$$

Note that if $q^d_{\text{tran}(l),e_l(t)}(t) - q^d_{\text{rec}(l),e_l(t)}(t) < \gamma^{e_l(t)}_l$, then $w_l(t) = 0$ and $\mu_l\big(\vec{P}(t)\big) = 0$ where $\vec{P}(t)$ is the solution of Equation (11), then $\big(q^d_{\text{tran}(l),e_l(t)}(t) - q^d_{\text{rec}(l),e_l(t)}(t)\big)\mu_l\big(\vec{P}(t)\big) = w_l(t)\mu_l\big(\vec{P}(t)\big)$; otherwise, $q^d_{\text{tran}(l),e_l(t)}(t) - q^d_{\text{rec}(l),e_l(t)}(t) \geq w_l(t)$, then $\big(q^d_{\text{tran}(l),e_l(t)}(t) - q^d_{\text{rec}(l),e_l(t)}(t)\big)\mu_l\big(\vec{P}(t)\big) \geq w_l(t)\mu_l\big(\vec{P}(t)\big)$. Combine with Equation (4) and Equation (10), we then have

$$\begin{aligned}\Delta(t) \leq& \frac{1}{K}\sum_{n,e}\Big[A^2_{n,e}(t) + 2\mu_{\max}A_{n,e}(t) + 2\mu^2_{\max}\Big] + \frac{1}{K}\sum_n\\ &\Big[\big(1+P_{\text{peak}}\big)^2 + \big(\eta^o_n\big)^2 + r^2_{\max} + 2\eta^o_n r_{\max}\Big] + V\sum_{n,e}\\ &R_{n,e}(t) + \sum_{n,e}\left[\frac{2}{K}q^d_{n,e}(t) - V\right]R_{n,e}(t) + \frac{2}{K}\sum_n \tilde{q}^b_n(t)\\ &\Big[I^o_n(t) + r_n(t) + M_n(t)\Big] - \frac{2}{K}\sum_l\Big[w_l(t)\mu_l\big(\vec{P}(t)\big) -\\ &\Big(\tilde{q}^b_{\text{tran}(l)}(t) + \tilde{q}^b_{\text{rec}(l)}(t)\Big)P_l(t)\Big]. \qquad (15)\end{aligned}$$

Lemma 1: Let S denote the set of activated links chosen by the *MPA* algorithm, then for $l \in S$, we have

$$\begin{aligned}\mu_l(\vec{P}) &= \log\left(1 + \frac{P_l}{\sigma_l + \sum_{i\in\mathcal{L}, i\neq l} h_{i,l}P_i}\right)\\ &\geq \mu_l(P_l) - c_l = \log\left(1 + \frac{P_l}{\sigma_l}\right) - c_l\end{aligned}$$

where $c_l = \frac{P_{\text{peak}}}{\sigma^2_l}\sum_{i\in S, i\neq l} HP_{\text{peak}}$ is some constant related to the threshold H of the crosstalk.

Proof: Note that $\log(1+x) \leq x$ for $x \geq 0$. Then

$$\begin{aligned}&\log\left(1 + \frac{P_l}{\sigma_l}\right) - \log\left(1 + \frac{P_l}{\sigma_l + \sum_{i\in\mathcal{L}, i\neq l} h_{i,l}P_i}\right)\\ &= \log\left(1 + \frac{\frac{P_l}{\sigma_l}\sum_{i\in\mathcal{L}, i\neq l} h_{i,l}P_i}{\sigma_l + P_l + \sum_{i\in\mathcal{L}, i\neq l} h_{i,l}P_i}\right)\\ &\leq \frac{\frac{P_l}{\sigma_l}\sum_{i\in\mathcal{L}, i\neq l} h_{i,l}P_i}{\sigma_l + P_l + \sum_{i\in\mathcal{L}, i\neq l} h_{i,l}P_i} \leq \frac{\frac{P_l}{\sigma_l}\sum_{i\in\mathcal{L}, i\neq l} h_{i,l}P_i}{\sigma_l + \sum_{i\in\mathcal{L}, i\neq l} h_{i,l}P_i}\\ &\leq \frac{P_{\text{peak}}}{\sigma^2_l}\sum_{i\in S, i\neq l} HP_{\text{peak}} = c_l,\end{aligned}$$

the last inequality is from the fact that the link in S has crosstalk less than H by the algorithm. ■

By Lemma 1, we have $\sum_l\Big[w_l(t)\mu_l\big(\vec{P}(t)\big) - \big(\tilde{q}^b_{\text{tran}(l)}(t) + \tilde{q}^b_{\text{rec}(l)}(t)\big)P_l(t)\Big] = \sum_{l\in S(t)}\Big[w_l(t)\mu_l\big(\vec{P}(t)\big) - \big(\tilde{q}^b_{\text{tran}(l)}(t) + \tilde{q}^b_{\text{rec}(l)}(t)\big)P_l(t)\Big] \geq \sum_{l\in S(t)}\Big[w_l(t)\mu_l\big(P_l(t)\big) - \big(\tilde{q}^b_{\text{tran}(l)}(t) + \tilde{q}^b_{\text{rec}(l)}(t)\big)P_l(t)\Big] - \sum_{l\in S(t)}\big(\frac{V}{2} + A_{\max}\big)c_l$.

Observe Equation (15), it is apparent that *MRC* is trying to minimize the term $\sum_{n,e}\left[\frac{2}{K}q^d_{n,e}(t) - V\right]R_{n,e}(t)$, and *MPA* is trying to maximize the value of the term $\sum_{l\in S(t)}\Big[w_l(t)\mu_l\big(P_l(t)\big) - \big(\tilde{q}^b_{\text{tran}(l)}(t) + \tilde{q}^b_{\text{rec}(l)}(t)\big)P_l(t)\Big]$. By *MRC*, we have for any $n, e \in \mathcal{N}$, $n \neq e$ that $\left[\frac{2}{K}q^d_{n,e}(t) - V\right]R_{n,e}(t) \leq \left[\frac{2}{K}q^d_{n,e}(t) - V\right]R^*_{n,e}(t)$ when $A_{n,e}(t) \leq W$. Then,

$$\begin{aligned}&\left[\frac{2}{K}q^d_{n,e}(t) - V\right]R_{n,e}(t)\\ &\leq \left[\frac{2}{K}q^d_{n,e}(t) - V\right]R^*_{n,e}(t) +\\ &\left(\frac{2}{K}q^d_{n,e}(t) - V\right)\big(R_{n,e}(t) - R^*_{n,e}(t)\big)I_{[A_{n,e}(t)>W]}\\ &\leq \left[\frac{2}{K}q^d_{n,e}(t) - V\right]R^*_{n,e}(t) + VA_{n,e}(t)I_{[A_{n,e}(t)>W]},\end{aligned}$$

the last inequality can be obtained simply by breaking into cases. Since the optimal solution for Problem (A) may not be unique, we let $\mathcal{P}^*$ be the optimal solution set and $\vec{P}^* \in \mathcal{P}^*$ be any optimal solution, for Problem (A). In time slot t, let $\vec{P}^m(t)$ be the value that maximize the unconstrained objective function $\sum_{l\in S(t)}\Big[w_l(t)\mu_l\big(P_l(t)\big) - \big(\tilde{q}^b_{\text{tran}(l)}(t) + \tilde{q}^b_{\text{rec}(l)}(t)\big)P_l(t)\Big]$. Only when $\vec{P}^m(t), \vec{P}^*(t) \notin \Pi(t)$, we may have $\sum_{l\in S(t)}\Big[w_l(t)\mu_l\big(P_l(t)\big) - \big(\tilde{q}^b_{\text{tran}(l)}(t) + \tilde{q}^b_{\text{rec}(l)}(t)\big)P_l(t)\Big] \leq \sum_{l\in S(t)}\Big[w_l(t)\mu_l\big(P^*_l(t)\big) - \big(\tilde{q}^b_{\text{tran}(l)}(t) + \tilde{q}^b_{\text{rec}(l)}(t)\big)P^*_l(t)\Big]$. When $\vec{P}^m(t), \vec{P}^*(t) \notin \Pi(t)$, *MPA* will drain at least one battery in order to maximize $\sum_{l\in S(t)}\Big[w_l(t)\mu_l\big(P^*_l(t)\big) - \big(\tilde{q}^b_{\text{tran}(l)}(t) + \tilde{q}^b_{\text{rec}(l)}(t)\big)P^*_l(t)\Big]$ within $\Pi(t)$ which is a concave objective function. Under this situation, we must have $I^o_n(t) = 1$ for some $n \in \mathcal{N}$ since $r_n(t-1) > 0$, $\forall n \in \mathcal{N}$. Further, note that

$\mu_l\big(P_l^*(t)\big) \geq \mu_l\big(\vec{P}^*(t)\big),\ \forall l \in \mathcal{L}$, we then have

$$\Delta(t) \leq \frac{1}{K}\sum_{n,e}\Big[A_{n,e}^2(t) + 2\mu_{\max}A_{n,e}(t) + 2\mu_{\max}^2\Big] + \frac{1}{K}\sum_{n}\Big[\big(1+P_{\text{peak}}\big)^2 + \big(\eta_n^o\big)^2 + r_{\max}^2 + 2\eta_n^o r_{\max} + \big(KV + 2A_{n,e}(t)\big)\big(2+\mu_{\max}\big)I_n^o(t)\Big] + V\sum_{n,e}R_{n,e}(t) + \frac{1}{K}\sum_{n,e}\Big[\frac{2}{K}q_{n,e}^d(t) - V\Big]R_{n,e}^*(t) - \frac{2}{K}\sum_{n,e}\sum_{l\in\Omega_n}\Big(q_{\text{tran}(l),e_l(t)}^d(t) - q_{\text{rec}(l),e}^d(t) - \gamma\Big)\mu_l^e\big(\vec{P}^*(t)\big)I_{[l\in S(t)]} + \frac{2}{K}\sum_n \tilde{q}_n^b(t)\Big[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t) + M_n(t)\Big] + \sum_{l\in S(t)}\big(V + \frac{2}{K}A_{\max}\big)c_l + V\sum_{n,e}A_{n,e}(t)I_{[A_{n,e}(t)>W]}$$

$$\leq \frac{1}{K}\sum_{n,e}\Big[A_{n,e}^2(t) + 2\big(A_{n,e}(t)+\gamma\big)\mu_{\max} + 2\mu_{\max}^2\Big] + \frac{1}{K}\sum_n\Big[\big(1+P_{\text{peak}}\big)^2 + \big(\eta_n^o\big)^2 + r_{\max}^2 + 2\eta_n^o r_{\max} + \big(KV + 2A_{n,e}(t)\big)\big(2+\mu_{\max}\big)I_n^o(t)\Big] + \frac{2}{K}\sum_n \tilde{q}_n^b(t)M_n(t) + V\sum_{n,e}\big(R_{n,e}(t) - \frac{R_{n,e}^*(t)}{K}\big) + \frac{2}{K}\sum_{n,e}q_{n,e}^d(t)\Big[\frac{R_{n,e}^*(t)}{K} + \Big(\sum_{l\in\Theta_n} - \sum_{l\in\Omega_n}\Big)\mu_l^e\big(\vec{P}^*(t)\big)I_{[l\in S(t)]}\Big] + \sum_{l\in S(t)}\big(V + \frac{2}{K}A_{\max}\big)c_l + \frac{2\sum_n\tilde{q}_n^b(t)}{K}\Big[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t)\Big] + V\sum_{n,e}A_{n,e}(t)I_{[A_{n,e}(t)>W]}. \tag{16}$$

Lemma 2:

$$\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_n \tilde{q}_n^b(t)\Bigg[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t)\Bigg] \leq O(\frac{1}{V}).$$

Proof: Note that

$$\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) \leq \liminf_{T\to\infty}\frac{1}{T}\Big[q_n^b(0) + \sum_{t=0}^{T-1}r_n(t)\Big] = \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}r_n(t).$$

Construct an auxiliary queue with the following evolution:

$$\bar{q}_n^{b*}(t+1) = \Big(\bar{q}_n^{b*}(t) - r_n(t) - \epsilon\Big)^+ + \sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t),$$

where $\epsilon > 0$ can be arbitrarily small. By Lemma 6, $\bar{q}_n^{b*}(t)$ is strongly stable. By multiplying $\tilde{q}_n^b(t)$ for both sides of the inequality $\bar{q}_n^{b*}(t+1) \geq \bar{q}_n^{b*}(t) - r_n(t) - \epsilon + \sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t)$ and rearranging terms, we obtain $\tilde{q}_n^b(t)\Big[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t)\Big] \leq \tilde{q}_n^b(t)\Big[\bar{q}_n^{b*}(t+1) - \bar{q}_n^{b*}(t) + \epsilon\Big]$. By summing from 0 to $T-1$, dividing by T and taking $\liminf_{T\to\infty}$, we have

$$\begin{aligned}
&\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_n\tilde{q}_n^b(t)\Bigg[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t)\Bigg]\\
&= \frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_n\bar{q}_n^{b*}(t)(\tilde{q}_n^b(t-1) - \tilde{q}_n^b(t))\\
&\quad + \frac{1}{V}\sum_n\liminf_{T\to\infty}\frac{\tilde{q}_n^b(T)\bar{q}_n^{b*}(T) - \tilde{q}_n^b(0)\bar{q}_n^{b*}(0)}{T}\\
&\quad + \liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_n\frac{\tilde{q}_n^b(t)}{V}\epsilon\\
&\leq \frac{1}{V}(1+P_{\text{peak}})\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_n\bar{q}_n^{b*}(t) + N\frac{\epsilon}{V}(\frac{KV}{2} + W)\\
&= O(\frac{1}{V}) + (\frac{NK}{2} + \frac{NW}{V})\epsilon,
\end{aligned}$$

since the average queue length of the auxiliary queue remains finite no matter how large the system parameter B_n^b becomes and is not related to the algorithmic parameters V, H and W. By letting $\epsilon \to 0$, we finish the proof. ■

Lemma 3:

$$\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_n\tilde{q}_n^b(t)M_n(t) \leq \sum_n O(\frac{(\frac{K}{2}V + W - B_n^b)^+}{V}).$$

Proof: Consider and $n \in \mathcal{N}$. Without loss of generality, let $q_n^b(0) = 0$. Observe Equation (10), we have the following cases:

i) if $\sum_{l\in\Omega_n\bigcup\Theta_n}P_l(t) \geq r_n(t)$, $I_n^o(t) = 0$ and $\tilde{q}_n^b(t) > 0$, then $M_n(t) = 0$, $q_n^b(t+1) \leq q_n^b(t)$ and $\tilde{q}_n^b(t+1) - \tilde{q}_n^b(t) \leq q_n^b(t) - q_n^b(t+1)$, i.e., even if $\tilde{q}_n^b(t)$ increases, the increment is no larger than the decrement of $q_n^b(t)$;

ii) if $\sum_{l\in\Omega_n\bigcup\Theta_n}P_l(t) < r_n(t)$, $I_n^o(t) = 0$, then if $\tilde{q}_n^b(t+1) > 0$, $\tilde{q}_n^b(t) - \tilde{q}_n^b(t+1) = r_n(t) - \sum_{l\in\Omega_n\bigcup\Theta_n}P_l(t) - M_n(t) - \eta_n^o + (\eta_n^o - \tilde{q}_n^b(t))^+ \geq q_n^b(t+1) - q_n^b(t) = r_n(t) - \sum_{l\in\Omega_n\bigcup\Theta_n}P_l(t) - M_n(t)$, i.e., the decrement of $\tilde{q}_n^b(t)$ is no less than the increment of $q_n^b(t)$, else if $\tilde{q}_n^b(t+1) = 0$, it goes to case iv);

iii) if $I_n^o(t) = 1$, then $q_n^b(t+1) = \min[r_n(t), B_n^b]$ by definition of discharging event Equation (3). If $M_n(t) > 0$, then this means $r_n(t) > B_n^b$ and $-r_n(t) + M_n(t) = -B_n^b$;

iv) if $\tilde{q}_n^b(t) = 0$, then $\tilde{q}_n^b(t)M_n(t) < 0$ anyway.

Combine the above discussion with Equation (13), we have that $\forall t \geq 0$, if $M_n(t) > 0$ and $\tilde{q}_n^b(t) > 0$, we must have

$\tilde{q}_n^b(t) \leq (\frac{KV}{2}+W)+\max[r_{\max},1]-B_n^b$. Thus,

$$\begin{aligned}&\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\tilde{q}_n^b(t)M_n(t)\\&\leq\frac{((\frac{KV}{2}+W)+\max[r_{\max},1]-B_n^b)^+r_{\max}}{V}\\&=O(\frac{(\frac{K}{2}V+W-B_n^b)^+}{V}).\quad\blacksquare\end{aligned}$$

Under the optimal policy, the amount of data destined to the same destination may be routed through different routes. We first logically decompose the flows that are differentiated by destination into flows that are differentiated by routes under this optimal policy. Let $\mathcal{F}$ denote the set of flows after the flow decomposition, then any flow $f\in\mathcal{F}$ has a fixed route. Let $e(f)$ denote the original flow from which flow f is decomposed, and $U_f(l)$ denote the one hop upstream link of link l in the route of flow f. Let $H^*=\{H_f^{l*}\}$ denote the routing matrix under the optimal policy, where $H_f^{l*}=1$ means the data of flow f is routed through link l. We further logically decompose the data queues at nodes into data queues at links under this optimal policy. Since the sum queue length of the decomposed queues equals the original queue length, we have

$$\begin{aligned}&\sum_{n,e}q_{n,e}^d(t)\left[\frac{R_{n,e}^*(t)}{K}+\left(\sum_{l\in\Theta_n}-\sum_{l\in\Omega_n}\right)\mu_l^e\left(\vec{P}^*(t)\right)I_{[l\in S(t)]}\right]\\&=\sum_{l,f}\left[\frac{1}{K}q_{\text{tran}(l),e(f)}^d(t)R_{\text{tran}(l),f}^*(t)-\left(q_{\text{tran}(l),e(f)}^d(t)-\right.\right.\\&\left.\left.q_{\text{rec}(l),e(f)}^d(t)\right)\mu_l^f\left(\vec{P}^*(t)\right)I_{[l\in S(t)]}\right]\\&=\sum_{l,f}q_{\text{tran}(l),e(f)}^d(t)\left[\frac{\sum_n R_{n,f}^*(t)H_f^{l*}}{K}-\mu_l^f\left(\vec{P}^*(t)\right)I_{[l\in S(t)]}-\right.\\&\left.\delta-\frac{\sum_n R_{n,f}^*(t)H_f^{U_f(l)*}}{K}+\mu_{U_f(l)}^f\left(\vec{P}^*(t)\right)I_{[U_f(l)\in S(t)]}+\delta\right]\\&\leq\sum_{l,f}q_{\text{tran}(l),e(f)}^d(t)\left(\bar{Q}_{l,f}^{d*}(t+1)-\bar{Q}_{l,f}^{d*}(t)\right)-\\&\sum_{l,f}q_{\text{tran}(l),e(f)}^d(t)\left(\bar{Q}_{U_f(l),f}^{d*}(t+1)-\bar{Q}_{U_f(l),f}^{d*}(t)\right)\end{aligned}\tag{17}$$

where $\delta>0$ and $\bar{Q}_{l,f}^{d*}(t),\ \forall l\in\mathcal{L},f\in\mathcal{F}$ has the following evolution:

$$\begin{aligned}\bar{Q}_{l,f}^{d*}(t+1)=&\left(\bar{Q}_{l,f}^{d*}(t)-\mu_l^f\left(\vec{P}^*(t)\right)I_{[l\in S(t)]}-\delta\right)^+\\&+\frac{1}{K}\sum_n R_{n,f}^*(t)H_f^{l*}.\end{aligned}\tag{18}$$

Lemma 4: If all the queues $\bar{Q}_{l,f}^{d*}(t),\ \forall l\in\mathcal{L},f\in\mathcal{F}$ with evolution Equation (18) are strongly stable, then $\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{n,e}q_{n,e}^d(t)\Big[\frac{R_{n,e}^*(t)}{K}+\big(\sum_{l\in\Theta_n}-\sum_{l\in\Omega_n}\big)\mu_l^e\big(\vec{P}^*(t)\big)I_{[l\in S(t)]}\Big]\leq O(\frac{W}{V})$

Proof: Since $-W\leq q_{n,e}^d(t)-q_{n,e}^d(t+1)\leq\mu_{\max},\ \forall t\geq 0,\ \forall n,e\in\mathcal{N}$, then

$$\begin{aligned}&\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\left[\sum_{l,f}q_{\text{tran}(l),e(f)}^d(t)\left(\bar{Q}_{l,f*}^d(t+1)-\bar{Q}_{l,f*}^d(t)\right)\right.\\&\left.-\sum_{l,f}q_{\text{tran}(l),e(f)}^d(t)\left(\bar{Q}_{U_f(l),f*}^d(t+1)-\bar{Q}_{U_f(l),f*}^d(t)\right)\right]\\&\leq\sum_{n,e}\left[\lim_{T\to\infty}\frac{q_{\text{tran}(l),e(f)}^d(T)\bar{Q}_{l,f*}^d(T)}{T}+\mu_{\max}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\right.\\&\bar{Q}_{l,f*}^d(t)+W\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\bar{Q}_{U_f(l),f*}^d(t)\\&\left.+\lim_{T\to\infty}\frac{q_{\text{tran}(l),e(f)}^d(0)\bar{Q}_{U_f(l),f*}^d(0)}{T}\right]\\&\leq\sum_{n,e}\left[\mu_{\max}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\bar{Q}_{l,f*}^d(t)+W\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\right.\\&\left.\bar{Q}_{U_f(l),f*}^d(t)\right].\end{aligned}\tag{19}$$

The average queue length of $\bar{Q}_{l,f}^{d*}(t)$ is not related to V, H and W. Combine with Equation (17), we obtain the result. $\blacksquare$

Now we only need to show that all the accumulative data queues $\bar{Q}_{l,f}^{d*},\ \forall l\in\mathcal{L},f\in\mathcal{F}$ are strongly stable. We know that all the data queues are strongly stable under the optimal policy queue evolution, then the queues are still strongly stable after flow and link decomposition. This further implies that the accumulative queues are strongly stable (note that the accumulative queues being stable does not imply each queue in the routes are stable), i.e., the queues with the following evolutions are strongly stable:

$$Q_{l,f}^{d*}(t+1)=\left(Q_{l,f}^{d*}(t)-\mu_l^f\left(\vec{P}^*(t)\right)\right)^++\sum_n R_{n,f}^*(t)H_f^{l*}.\tag{20}$$

Let $\bar{\mu}_l^{f*}=\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu_l^f\left(\vec{P}^*(t)\right)$ and $\lambda_l^{f*}=\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_n R_{n,f}^*(t)H_f^{l*}$. Without loss of generality, assume $\bar{\mu}_l^{f*}>0$. We know the optimal sensing rate is feasible, so $\lambda_l^{f*}<\bar{\mu}_l^{f*}$. Then for the normalized queues with the following evolutions, the arrival rate $\frac{\lambda_l^{f*}}{\bar{\mu}_l^{f*}}$ is also feasible:

$$q_{l,f}^{d*}(t+1)=\left(q_{l,f}^{d*}(t)-1\right)^++\frac{\lambda_l^{f*}}{\bar{\mu}_l^{f*}}.\tag{21}$$

Note that $I_{[l\in S(t)]}$ is a random variable which is independent of $\vec{P}^*(t)$. Thus, $I_{[l\in S(t)]}$ and $\mu_l^f\left(\vec{P}^*(t)\right)$ are independent and then uncorrelated, so

$$\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu_l^f\left(\vec{P}^*(t)\right)I_{[l\in S(t)]}=\bar{\mu}_l^{f*}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}I_{[l\in S(t)]}.\tag{22}$$

In order to show that all the queues with evolution Equation (18) are strongly stable, we need to show that the

normalized data queues under the *MPA* algorithm with the following queue evolutions are strongly stable:

$$\bar{q}^{d*}_{l,f}(t+1) = \left(\bar{q}^{d*}_{l,f}(t) - I_{[l\in S(t)]}\right)^+ + \frac{\lambda_l^{f*}}{K\bar{\mu}_l^{f*}}. \quad (23)$$

Lemma 5: If the arrival rate $\frac{\lambda_l^{f*}}{\bar{\mu}_l^{f*}}$ is feasible for queues with evolution Equation (21), then the queues with evolution Equation (23) are all strongly stable.

Proof: Similar to the idea of [20], we let $L(\vec{\bar{q}}^{d*}(t)) = \sum_{l,f} \bar{q}^{d*}_{l,f}(t)\left(\sum_{k\in l\bigcup \mathcal{E}_l,f} \bar{q}^{d*}_{k,f}(t)\right)$. Let $\mathcal{E}_l$ denote the set of links that will not be activated under *MPA* algorithm when link l is activated. Then

$$\begin{aligned}\Delta_{L(\vec{\bar{q}}^{d*})}(t) &= L(\vec{\bar{q}}^{d*}(t+1)) - L(\vec{\bar{q}}^{d*}(t)) \\ &\leq 2\sum_{l,f:\bar{q}^{d*}_{l,f}(t)\geq 1} \bar{q}^{d*}_{l,f}(t)\Bigg[\sum_f \sum_{k\in l\bigcup\mathcal{E}_l}\Bigg(\frac{\lambda_k^{f*}}{K\bar{\mu}_k^{f*}} \\ &\quad - I_{[k\in S(t)]}\Bigg)\Bigg] + L^2F^2\left(1+\left(\max_{l,f}\frac{\lambda_l^{f*}}{K\bar{\mu}_l^{f*}}\right)^2\right).\end{aligned}$$

Note that $\sum_{k\in l\bigcup\mathcal{E}_l} I_{[k\in S(t)]}\geq 1$ and $\sum_{k\in l\bigcup\mathcal{E}_l}\frac{\lambda_k^{f*}}{\bar{\mu}_k^{f*}} < K$ for any $f\in\mathcal{F}$. By summing from 0 to $T-1$, dividing by T and taking $\liminf_{T\to\infty}$ for the above equation, we have

$$\begin{aligned}&\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{l,f}\bar{q}^{d*}_{l,f}(t) \\ &\leq 1+\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{l,f:\bar{q}^{d*}_{l,f}(t)\geq 1}\bar{q}^{d*}_{l,f}(t) < \infty. \quad \blacksquare\end{aligned}$$

By Lemma 5, queues with evolution Equation (23) are all strongly stable, then $\frac{\lambda_l^{f*}}{K\bar{\mu}_l^{f*}} \leq \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} I_{[l\in S(t)]}$. Combine with Equation (22), we have

$$\begin{aligned}&\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\left(\frac{1}{K}\sum_n R^*_{n,f}(t)H_f^{l*}\right) \\ &< \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\Big(\mu_l^f(\vec{P}^*(t))I_{[l\in S(t)]}+\delta\Big). \quad (24)\end{aligned}$$

The following lemma gives the sufficient condition for strong stability under sample mean argument.

Lemma 6: For any queue with evolution $q(t+1) = \big(q(t)-\mu(t)\big)^+ + \lambda(t)$, if $\lambda(t)\leq\lambda_{\max}$, $\forall t\geq 0$ and $\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\lambda(t) < \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu(t)$, then $q(t)$ is strongly stable, i.e., $\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}q(t)<\infty$.

Proof: For any time t, either there exists a time $J(t)\leq t$ such that $q(i+1) = q(i)-\mu(i)+\lambda(i)$, $\forall i\in[J(t),t]$, or $q(t+1)=\lambda(t)$. Further, $\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}(\lambda(t)-\mu(t))<0$ implies that there exists T_0 such that $\sum_{t=T_1}^{T_2}(\lambda(t)-\mu(t))<\infty$, $\forall T_1,T_2\geq T_0$. Then,

$$\begin{aligned}&\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}q(t) \\ &\leq \limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\left(\lambda_{\max}+\sum_{i=J(t)}^{t}\Big(\lambda(i)-\mu(i)\Big)\right) \\ &\leq \lambda_{\max}+\limsup_{T\to\infty}\frac{1}{T}\sum_{t=T_0}^{T-1}\sum_{i=J(t)}^{t}\Big(\lambda(i)-\mu(i)\Big)<\infty. \quad \blacksquare\end{aligned}$$

Let $\sigma=\min_l\sigma_l$, then $\sum_{l\in S(t)}c_l\leq\left(\frac{P_{\text{peak}}L}{\sigma}\right)^2 H$. Combine with Lemma 2, Lemma 3, Lemma 4, Lemma 6, by summing from 0 to $T-1$, dividing by T and V, taking $\liminf_{T\to\infty}$ for Equation (16), we get

$$\begin{aligned}&\sum_n\left\{\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_e R_{n,e}(t)\right\} \\ &\geq \sum_n\Bigg\{\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_e\frac{R^*_{n,e}(t)}{K}-\eta_n^o\big(2+\mu_{\max}\big)- \\ &\quad O\Big(\frac{(\frac{K}{2}V+W-B_n^b)^+}{V}\Big)\Bigg\}-\left(\frac{P_{\text{peak}}L}{\sigma}\right)^2 H-O(\frac{W}{V}) \\ &\quad -\sum_{n,e}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}A_{n,e}(t)I_{[A_{n,e}(t)>W]}.\end{aligned}$$

Since $\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}A_{n,e}(t)\leq\lambda_{\max}<\infty$ and

$$\begin{aligned}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}A_{n,e}(t) &= \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}A_{n,e}(t)I_{[A_{n,e}(t)\leq W]} \\ &\quad +\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}A_{n,e}(t)I_{[A_{n,e}(t)>W]}.\end{aligned}$$

By Monotone Convergence Theorem, we have $\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}A_{n,e}(t)I_{[A_{n,e}(t)>W]}\to 0$ as $W\to\infty$, which implies Equation (14). $\blacksquare$

C. Proof of Theorem 3

Consider any link $l\in\mathcal{L}$, for $\forall k\in\mathcal{E}_l$, we have $h_{k,l}=\frac{g_{\text{tran}(k),\text{rec}(l)}}{g_{\text{tran}(l),\text{rec}(l)}}=\left(\frac{d(\text{tran}(k),\text{rec}(l))}{d_l}\right)^{-\beta}>H$, or $h_{l,k}=\frac{g_{\text{tran}(l),\text{rec}(k)}}{g_{\text{tran}(k),\text{rec}(k)}}=\left(\frac{d(\text{tran}(l),\text{rec}(k))}{d_k}\right)^{-\beta}>H$, i.e., $d(\text{tran}(k),\text{rec}(l))<d_lH^{-\frac{1}{\beta}}$, or $d(\text{tran}(l),\text{rec}(k))<d_kH^{-\frac{1}{\beta}}$.

As shown in Figure. 5, any link whose transmitter is in the $d_lH^{-\frac{1}{\beta}}$ neighborhood of $\text{rec}(l)$, and any link k whose receiver is within the distance $d_kH^{-\frac{1}{\beta}}$ of $\text{tran}(l)$ are conflicting with link l. We need to find an upper bound for the number of nonconflicting links in $\mathcal{E}_l$. Let $r_l=d_lH^{-\frac{1}{\beta}}$, $\forall l\in\mathcal{L}$, $r=d_{\min}H^{-\frac{1}{\beta}}$ and $R=d_{\max}H^{-\frac{1}{\beta}}$.

I) We first consider the $d_lH^{-\frac{1}{\beta}}$ neighborhood of $\text{rec}(l)$.

Fact 1: From Figure. 5, it is apparent that with a fixed H, if $\text{tran}(i)$ and $\text{tran}(j)$ are both within $\mathcal{E}_l$, as $d(\text{tran}(i),\text{rec}(l))$ and $d(\text{tran}(j),\text{rec}(l))$ increase, the angle θ gets smaller by placing $\text{tran}(i)$ and $\text{tran}(j)$ closer without letting i conflict with j.

Thus, we can just assume $d_l = d_{\max}$ and

Fact 2: If $r_i < d_i$ and $r_j < d_i$, i.e., $H \geq 1$, the transmitters in the $d_l H^{-\frac{1}{\beta}}$ neighborhood of rec(l) can be placed very dense and the upper bound for the number of nonconflicting links in this area is $2\pi\rho d_{\max}^2 H^{-\frac{2}{\beta}} \leq 2\pi\rho d_{\max}^2$.

We then consider the case when $H < 1$, from Figure. 5, it is easy to have the following

$$(d_i + R - R\cos\theta_R)^2 + R^2\sin^2\theta_R = r_i^2 = \left(\frac{Rd_i}{d_{\max}}\right)^2.$$

Then $\cos\theta_R = 1 - \frac{R^2 - d_{\max}^2}{d_{\max}^2}\left(\frac{R+d_i}{2R} + \frac{R}{2(R+d_i)} - 1\right)$, and it is minimized by letting $d_i = d_{\min}$. Thus,

$$\begin{aligned}\cos\theta_R &= 1 - \frac{d_{\min}^2}{2d_{\max}^2}\frac{R^2 - d_{\max}^2}{R(d_{\min}+R)} \\ &= 1 - \frac{d_{\min}^2}{2d_{\max}^2}\left(1 - \frac{d_{\max}}{d_{\min}H^{-\frac{1}{\beta}}} + \frac{\frac{d_{\max}^2 - d_{\min}^2}{d_{\min}}}{d_{\min} + d_{\max}H^{-\frac{1}{\beta}}}\right),\end{aligned}$$

which is monotonically decreasing to $1 - \frac{d_{\min}^2}{2d_{\max}^2}$ as $H \to 0$. Thus, $\theta_{\min} = \arccos\left(1 - \frac{d_{\min}^2}{2d_{\max}^2}\right)$ and there are at most $\lfloor\frac{2\pi}{\theta_{\min}}\rfloor$ transmitters can be placed along the outmost circle of rec(l)'s neighborhood. By *Fact 1*, in the inner circles, the radius gets smaller, and the minimum angle for placement increases. Therefore, $\lfloor\frac{2\pi}{\theta_{\min}}\rfloor$ is also an upper bound for number of transmitters placed along all the inner circles, then there are at most $\xi\left\lfloor\frac{2\pi}{\arccos\left(1 - \frac{d_{\min}^2}{2d_{\max}^2}\right)}\right\rfloor$ nonconflicting links in the $d_l H^{-\frac{1}{\beta}}$ neighborhood of rec(l) when $H < 1$.

II) We now consider the conflicting links induced by tran(l). When $H \geq 1$, *Fact 2* is also valid here. So we consider $H < 1$. From Figure. 5, it is easy to see that placing a receiver of a link with length $d_{\max}$ and a receiver of a link with length $d_{\min}$ alternatively makes the angle larger. So, we only need to consider placing all receivers of link with length $d_{\max}$ or placing all receivers of link with length $d_{\min}$. Further, the conflicting distance to tran(l) is determined by the length of other links, so we only need to take care of one circle. Similar to the calculation in I), we have $\cos\varphi_R = \frac{1}{2} + \frac{d_{\max}}{2R} = \frac{1+H^{\frac{1}{\beta}}}{2} = \frac{1}{2} + \frac{d_{\min}}{2r} = \cos\varphi_r$ which is monotonically decreasing to $\frac{1}{2}$ as $H \to 0$. Thus, $\theta_{\min} = \frac{\pi}{3}$ and there are at most 6 receivers can be placed around tran(l).

Combining I) and II), we finish the proof. ■

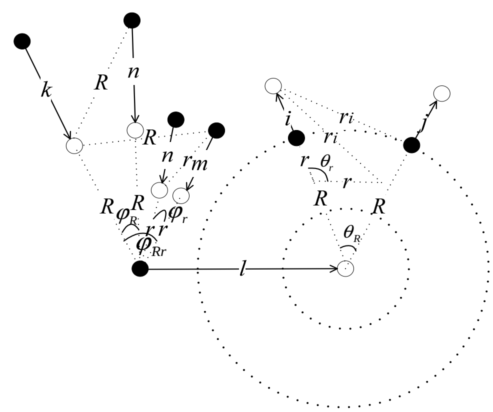

Fig. 5. Nonconflicting Links in $\mathcal{E}_l$